

# Method for Robust Spike Sorting with Overlap Decomposition

Guang-Li Wang and Pei-Ji Liang\*

**Abstract**—Spike sorting is the mandatory first step in analyzing multi-unit recording signals for studying information processing mechanisms within the nervous system. Extracellular recordings usually contain overlapped spikes produced by a number of neurons adjacent to the electrode, together with background noise having unknown properties. In the present study, a robust method to deal with these problems is proposed. The method employs an automatic overlap decomposition technique based on the relaxation (RELAX) algorithm that requires simple fast Fourier transforms (FFT's). The performance of the presented system was compared with that of a previously published method and tested at various signal-to-noise ratio (SNR) levels based on synthetic data that were generated from real data.

## I. INTRODUCTION

The detection and classification of neural spike activity from multi-unit recordings in the presence of background noise with unknown properties, a problem commonly referred to as spike sorting, is the mandatory first step in studying information processing mechanisms within the nervous system [1].

Methods of spike sorting have been extensively studied during the past decades and a large number of techniques and related problems have been summarized [2], but their application is severely limited by some unsolved problems. Firstly, a priori assumption of Gaussian distribution was usually made in these algorithms, but it cannot accurately capture the statistical characteristics of the background noise [3]. Secondly, spikes fired by different neurons will overlap temporally and produce distorted waveforms that conventional methods can hardly deal with. Recently, some algorithms considering the overlapping problem have been proposed [4], [5], but these methods were all with the restraint that the background noise properties were known a priori. In addition, several methods have been developed for spike sorting without modeling background noise as Gaussian, but their performance for overlap decomposition was not presented [6], [7].

In the present paper, an automatic method for dealing with these problems simultaneously is developed. Firstly, spike events are detected using a nonlinear energy operator [8], after the extracellular recordings are filtered (pass-band 100Hz~3kHz). Secondly, the features of spike waveforms

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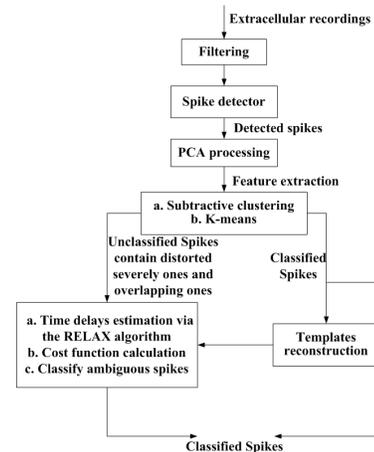


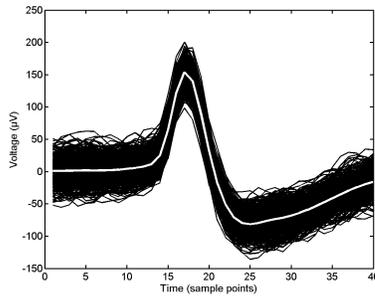
Fig. 1. Block diagram of the overall spike sorting system

are extracted based on principal component analysis (PCA) and are used to automatically reconstruct templates. Finally, in order to identify those ambiguous waveforms that contain overlapping spikes and severely distorted spikes, model analysis is performed and cost functions are calculated for all possible templates or combinations of spike waveforms in the frequency domain. In order to minimize the cost function that contains a number of unknown parameters, an approach based on a decoupled parameter estimation algorithm - the relaxation (RELAX) algorithm [9], which requires simple fast Fourier transforms (FFT's), is implemented. The spike template or the combination of spike templates, which is corresponding to the minimum of the minimized cost functions for all possible combinations, is assigned as the best fitting to the waveform being examined. This process is robust to background noise, because no assumptions about noise properties are requested. Fig. 1 shows the presented system.

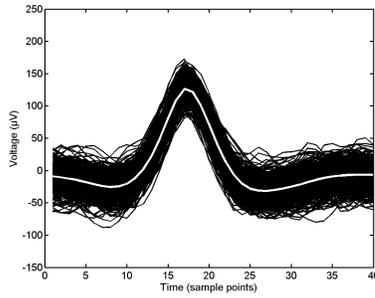
## II. DATA GENERATION

Synthetic data have some advantages over real data in evaluating the performance of algorithms, since the exact information such as the number of the spike classes and the firing times of each spike can be known in advance.

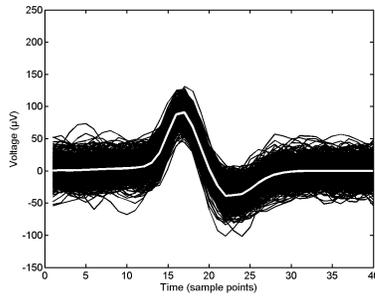
In this study, synthetic data were constructed using three spike waveforms recorded from chicken retinal ganglion cells [11] together with a segment of real background noise. Each template waveform lasted for 2 ms (40 sampling points, 16 points before and 23 points after the peak point). In order to construct the synthetic data, each template was superposed respectively with a segment of random background noise.



(a)



(b)

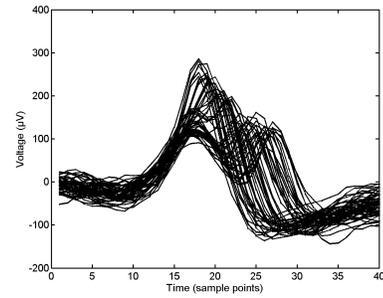


(c)

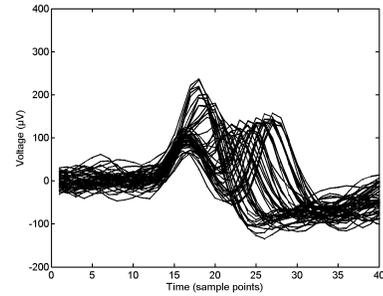
Fig. 2. Synthetic spike waveforms for three neurons (SNR = 2.5). Each group contains 500 spikes superposed with real background noise. Each spike consists of 40 sample points (2 ms). (a) Spikes generated by using template I and background noise. (b) Spikes generated by using template II and background noise. (c) Spikes generated by using template III and background noise.

This procedure was repeated for 500 times, which yielded totally 1500 spike waveforms distorted by background noise. Each two of the three templates were then superposed with each other with random time delays, together with a segment of random background noise. Fifty repeats resulted in a total number of 150 combinations. Finally, all of the three templates were superposed with random time delays and a segment of random background noise, with 50 repeats. All of the synthetic spike waveforms are shown in Fig. 2 and Fig. 3, where SNR was defined as

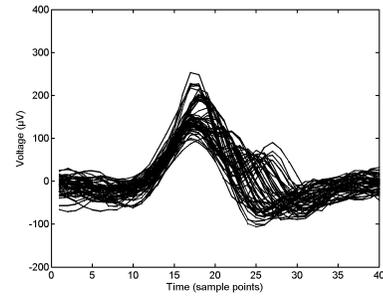
$$\text{SNR} = \frac{\text{root-mean-square value of action potential waveform}}{\text{root-mean-square value of pure noise segment}}$$



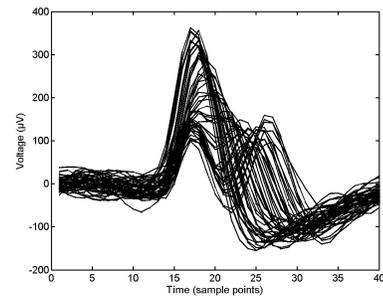
(a)



(b)



(c)



(d)

Fig. 3. Synthetic overlapping spikes (SNR = 2.5). Each group contains 50 overlapped spikes. (a) 50 overlapped spikes which are generated by superposing template I with delayed template II and background noise. (b) 50 overlapped spikes which are generated by superposing template I with delayed template III and background noise. (c) 50 overlapped spikes which are generated by superposing template II and delayed template III and background noise. (d) 50 overlapped spikes which are generated by superposing template I with delayed template II and III, and background noise.

### III. METHOD

#### A. Feature Extraction and Templates Reconstruction

In this study, PCA is used to extract feature from the data set, where  $N$  is the number of spike events. Here, the scores of the first three principal components serve as the feature for spike events classification. Then, a subtractive clustering technique [5] is employed to determine the number of the clusters and the initial values of the cluster centers by analyzing the feature. Since the estimate for the cluster centers based on the above-mentioned subtractive method might be somewhat biased, the K-means algorithm is further used to optimize the cluster centers. The spike events corresponding to the data points contained in the spherical areas whose radius is determined by the minimal of the distances between each pair of cluster centers, are regarded as classified spikes and others are considered as unclassified ones which will be classified or decomposed in the subsequent process. The template for each neuron is then reconstructed via averaging the spike waveforms that belong to each sphere.

#### B. Automatic Overlap Decomposition Based on the RELAX Algorithm via FFT's

The unclassified spike events that contain overlapping ones and severely distorted ones can be expressed as follows

$$y(n) = \sum_{m=1}^M s_m(n + \tau_m) + \varepsilon(n) \quad (1)$$

where  $M$  represents the possible number of neurons contributing to the waveform to be classified;  $N$  denotes the number of data samples included in each single waveform;  $s_m(n), m = 1, 2, \dots, M$  stands for the estimated template which is reconstructed based on the classified spikes;  $\tau_m, m = 1, 2, \dots, M$  reflects the relative time delay of the  $m$  th reconstructed template; and  $\varepsilon(n), n = 1, 2, \dots, N$  is the background noise with unknown properties. Let  $Y(\omega_k), S_m(\omega_k)$  and  $E(\omega_k)$  denote the Fourier transforms of  $y(n), s_m(n)$  and  $\varepsilon(n)$  at a certain frequency  $\omega_k$ , respectively. In this case, the time domain data model in (1) can alternatively be formulated in the frequency domain as

$$Y(\omega_k) = \sum_{m=1}^M S_m(\omega_k) e^{j\omega_k \tau_m} + E(\omega_k) \quad (2)$$

We form the following cost function based on  $M$  templates

$$C(\{\tau\}_{m=1}^M) = \sum_{k=1}^K \left| Y(\omega_k) - \sum_{m=1}^M S_m(\omega_k) e^{j\omega_k \tau_m} \right|^2 \quad (3)$$

Minimizing  $C(\{\tau\}_{m=1}^M)$  with respect to the unknown parameters  $\{\tau\}_{m=1}^M$  is a highly nonlinear optimization problem. Therefore, an approach based on the RELAX algorithm [9] is presented in this study.

Before presenting our approach to minimize the cost function, let us consider the following preparations. Let

$$Y_m(\omega_k) = Y(\omega_k) - \sum_{i=1, i \neq m}^M S_i(\omega_k) e^{j\omega_k \hat{\tau}_i} \quad (4)$$

and

$$\mathbf{g}_m = \sum_{k=1}^K |Y_m(\omega_k) - S_m(\omega_k) e^{j\omega_k \tau_m}|^2 \quad (5)$$

where  $\{\hat{\tau}_i\}_{i=1, i \neq m}^M$  are assumed to be given. In order to estimate  $\tau_m$  based on the minimized cost function (5), we can express the problem as

$$\min_{\tau_m} (\mathbf{g}_m) = \min_{\tau_m} \sum_{k=1}^K |Y_m(\omega_k) - S_m(\omega_k) e^{j\omega_k \tau_m}|^2 \quad (6)$$

Equation (6) can be rewritten as

$$\hat{\tau}_m = \arg \min_{\tau_m} \sum_{k=1}^K |Y_m(\omega_k) - S_m(\omega_k) e^{j\omega_k \tau_m}|^2 \quad (7)$$

The solution to the minimization problem can be found via the following

$$\hat{\tau}_m = \arg \max_{\tau_m} \left\{ \operatorname{Re} \sum_{k=1}^K [Y_m(\omega_k) S_m^*(\omega_k) e^{-j\omega_k \tau_m}] \right\} \quad (8)$$

Hence,  $\hat{\tau}_m, m = 1, 2, \dots, M$  is obtained as the location of the dominant peak of  $\operatorname{Re} \sum_{k=1}^K [Y_m(\omega_k) S_m^*(\omega_k) e^{-j\omega_k \tau_m}]$ , which can be efficiently computed by applying the FFT, using the sequence  $[Y_m(\omega_k) S_m^*(\omega_k)]$  padded with zeros as input.

With the above preparations, we can describe the steps of the approach based on the RELAX algorithm as follows

Step 1: Let  $M = 1$ . Obtain  $\hat{\tau}_1$  from  $Y(\omega_k)$  as described above, and calculate the cost function by using (3).

Step 2: Let  $M = 2$ . Compute  $Y_2(\omega_k)$  with (4) by using  $\hat{\tau}_1$ . Obtain  $\hat{\tau}_2$  from  $Y_2(\omega_k)$  by using (8). Next, compute  $Y_1(\omega_k)$  with (4) by using  $\hat{\tau}_2$ , and re-determine  $\hat{\tau}_1$  from  $Y_1(\omega_k)$  by using (8).

Iterate Step 2 until “practical convergence” is achieved, and calculate the cost function by using (3).

Step 3: Let  $M = 3$ . Compute  $Y_3(\omega_k)$  with (4) by using  $\{\hat{\tau}_i\}_{i=1,2}$ . Obtain  $\hat{\tau}_3$  from  $Y_3(\omega_k)$  by using (8). Next, compute  $Y_1(\omega_k)$  by using  $\{\hat{\tau}_i\}_{i=2,3}$  and re-determine  $\hat{\tau}_1$  from  $Y_1(\omega_k)$ . Then compute  $Y_2(\omega_k)$  by using  $\{\hat{\tau}_i\}_{i=1,3}$  and re-determine  $\hat{\tau}_2$  from  $Y_2(\omega_k)$ .

Iterate Step 3 until “practical convergence” is achieved, and finally, calculate the cost function by using (3).

Remaining Steps: Continue similarly until  $M$  is equal to the pre-determined number of reconstructed templates  $T$ .

The “practical convergence” in the iterations of the above approach based on the RELAX algorithm can be determined by checking the relative change of the cost function in (3) between the two consecutive iterations.

Using the above method, the algorithm yields values of the minimized cost functions for all the possible templates or template combinations. The template or template combination corresponding to the minimum of all the minimized cost functions reflects the optimal classification of the spike event examined.

TABLE I

TEST RESULTS OF SYNTHETIC DATA USING THE PRESENTED SYSTEM

Computation Results Template	Class I	Class II	Class III	Class I+II	Class I+III	Class II+III	Class I+II+III	Correct Rate
Template I	487	2	1	9	0	1	0	97.4%
Template II	4	483	6	0	0	7	0	96.6%
Template III	4	3	493	0	0	0	0	98.6%
Template I+II	0	0	0	41	7	0	2	82%
Template I+III	5	0	1	0	42	2	0	84%
Template II+III	0	4	2	0	0	44	0	88%
Template I+II+III	0	0	0	5	6	1	38	76%

TABLE II

TEST RESULTS OF SYNTHETIC DATA USING ATIYA'S METHOD

Computation Results Template	Class I	Class II	Class III	Class I+II	Class I+III	Class II+III	Class I+II+III	Correct Rate
Template I	427	21	33	11	3	2	3	85.4%
Template II	11	429	44	2	6	6	2	85.8%
Template III	21	54	410	2	3	5	5	82.0%
Template I+II	6	3	0	31	5	1	4	62%
Template I+III	9	2	7	1	27	1	3	54%
Template II+III	2	3	2	7	3	30	3	60%
Template I+II+III	3	2	1	8	6	1	29	58%

#### IV. RESULTS AND DISCUSSION

Firstly, our method was applied to the synthetic data (SNR = 2.5) described in DATA GENERATION section. The results are summarized in Table I. As a comparison, a previously published method [4] was also applied to the same synthetic data. The results are listed in Table II. It is shown that the proposed system has a better performance, whether in coping with simple waveforms or overlapping waveforms.

Secondly, the presented method's performance was tested at various SNR levels. The SNR was tuned by amplifying the real background noise by a desired factor. In Fig. 4, the overall percentage of correct classification is plotted versus SNR level. The results show that the correct classification ratio of all simple spikes is over 95%, while the correct decomposition ratio of overlapping spikes is over 80%, under the situation that SNR is over 2.4, which is actually the situation we mostly encountered in our work with real experimental recordings.

In our method, it is true that the performance of PCA will decrease when SNR reduces. However, the templates can be properly estimated using those data that were in vicinities of the cluster centers being defined by spherical boundaries, so as to reduce the distortions that might be caused by the background noise. The results show that it is feasible to use this proposed method, without any a priori knowledge about the background noise.

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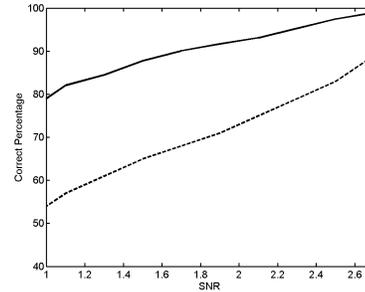


Fig. 4. The rate of correct classification for single spikes (solid curve) and overlap (dashed curve) decomposition under various SNR levels.

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